2)

$$\begin{cases} F'(x) = f(x) \ ; \ \forall x \in [1,+\infty[\\ F(1) = 0 \end{cases}$$

On a : $F(x) = \int_1^x f(t)dt$

$f(t) \le \frac{Lnt}{t}$; $\forall t \in [1,x]$

$\Rightarrow \int_1^x f(t)dt \le \int_1^x \frac{Lnt}{t} dt$

$\Rightarrow F(x) \le \left[\frac{1}{2}(Lnt)^2\right]_1^x$

$\Rightarrow F(x) \le \frac{1}{2}(Lnx)^2$

$\forall x \in [1,+\infty[$

EX 22 :

$x \in [0,+\infty[$

$f(x) = Ln\left(x+\sqrt{1+x^2}\right)$

1)

a) $f'(x) = \frac{1+\frac{2x}{2\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}$

$f'(x) = \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}\left(x+\sqrt{1+x^2}\right)} = \frac{1}{\sqrt{1+x^2}}$

b) $\int_0^1 \frac{dt}{\sqrt{1+t^2}} = [f(t)]_0^1 = Ln\left(1+\sqrt{2}\right)$

2) $f'(x) > 0$

x	0		$+\infty$
f'(x)		+	
f(x)	0	↗	$+\infty$

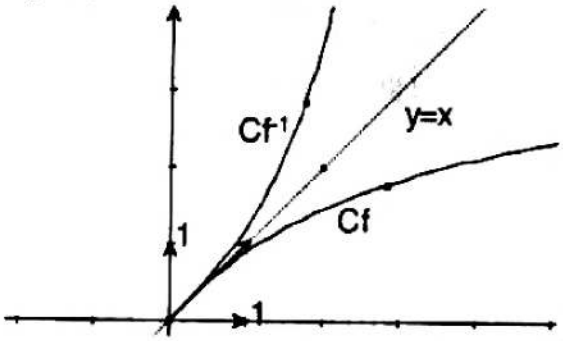
$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} Ln\left(\underbrace{x+\sqrt{1+x^2}}_{+\infty}\right) = +\infty$

$* \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{Ln\left[x\left(1+\sqrt{x+\frac{1}{x^2}}\right)\right]}{x}$

$= \lim_{x\to+\infty} \underbrace{\frac{Lnx}{x}}_{0} + \underbrace{\frac{Ln\left(1+\sqrt{1+\frac{1}{x^2}}\right)}{x}}_{0} = 0$

B.I de direction parabolique celle de $(o,\vec{i})$

$* f'(0) = 1$



3)

a) f est continue et strictement croissante sur $[0,+\infty[$ elle réalise donc une bijection de $[0,+\infty[$ sur $J = [0,+\infty[$

b) $(Cf-1) = S_\Delta(Cf)$

$\Delta : y = x$

c) $I = \int_0^1 Ln\left(x+\sqrt{1+x^2}\right).dx$

$$\begin{cases} U'(x) = 1 \xrightarrow{p} U(x) = x \\ V(x) = f(x) \xrightarrow{d} V'(x) = \frac{1}{\sqrt{1+x^2}} \end{cases}$$

$I = [x.f(x)]_0^1 - \int_0^1 \frac{x}{\sqrt{1+x^2}} dx$

$I = f(1) - \left[\sqrt{1+x^2}\right]_0^1$

$I = Ln\left(1+\sqrt{2}\right) - \left(\sqrt{2}-1\right)$

$\boxed{I = Ln\left(1+\sqrt{2}\right)+1-\sqrt{2}}$

Qarray.tn -Agra Blech- Qarray.tn